

DEVELOPMENT AND VALIDATION OF A COUPLED EULERIAN LAGRANGIAN FINITE ELEMENT ICE SCOUR MODEL

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ABSTRACT

Pipelines in the arctic offshore must be installed and buried below the seabed to avoid direct contact, and to mitigate the effects of strains induced by soil displacement below the ice keel scour depth. A three-dimensional (3D) Finite Element (FE) model which utilizes the Coupled Eulerian Lagrangian (CEL) formulation has been developed to provide direct and explicit estimation of pipe stresses and strains. The CEL formulation is novel, and no published work has attempted to explore its capabilities and potential for ice scour modeling to date. The developed model will be helpful in solving some of the uncertainty regarding pipeline burial depth, potentially resulting in major trenching cost savings.

In order to gain confidence in this numerical modeling technique, a systematic validation effort was carried out, whereby numerical predictions of subgouge displacements were compared with measured data from centrifuge tests and other published empirical and numerical data. Sensitivity analyses were then performed to investigate the effect of the scouring keel geometry, depth, and attack angle on the induced subgouge soil displacements. Preliminary conclusions were drawn and presented in this paper.

INTRODUCTION

The world demand of oil and gas is growing at an ever increasing rate, and as a result, there is a demand to explore new areas for more petroleum production. The arctic region is one of the remaining unexplored areas where such exploration still can be done. According to the US Geological Survey, the arctic region, mostly offshore, holds as much as 25 percent of the world's untapped hydrocarbon reserves, however, much of the reserve is lying under seasonal or year-round sea ice. This means additional challenges for design, construction, and operation of offshore installations. A compiled list of general arctic challenges may be found in Abdalla et al. [Ref. 2].

Among many environmental and geotechnical hazards pertaining to the design of marine pipelines, ice scouring (or ice gouging) is believed to be a major threat to the safety of pipelines. The ice gouging process may be summarized as follows: The phenomenon starts by wind and current forces piling up ice and sea ice into large masses called ice ridges. These ice ridges have a keel that extends below the water surface and it moves with the ice sheets and packed ice. When pushed towards shallower water with depths less than the keel draft, ice masses cut gouges into the seabed. Types of ice masses vary from place to place within the arctic region [Ref. 15]. If happened to be in the ice gouging track, pipelines may not be able to withstand the ice contact forces, thus, pipelines need to be buried below predicted extreme ice keel scour depths for protection.

A variety of ice gouging scenarios can be considered depending on the acting environmental forces, characteristics of ice features, and seabed conditions [Ref. 12]. Field evidence indicates that most scour (or gouge) features are uniform in cross section over long distances, and the seabed is often almost horizontal, implying that a simple steady state scouring condition may represent the most relevant condition in the practice [Ref. 16].

In a recent publication, Konuk et al. [Ref. 8] reported that gouges of different patterns have been observed in different parts of the arctic. Long gouges with the same uniform cross sectional profiles were observed in the Beaufort Sea, and are likely generated by a steady state gouging process. On the other hand, gouges with shorter average lengths were observed in the Eastern Barents Sea, and indicate that a steady state process is not always attained.

Ice scour is a complicated phenomenon which involves interaction between hydrodynamic forces, an ice feature, soil medium, and is further complicated by the presence of a pipeline. The parameter of interest to us in this complicated

process is the displacement of soil mass during scouring. If the pipeline is located in a zone of large soil displacement, it is likely to be damaged by ice scour. Unless the soil is very soft and expected to flow around the pipeline, the buried pipeline is generally considered a flexible structure, and will be carried by the movement of the soil. Stresses induced in seabed soil during scour are of less importance because these are limited to the soil strength, and offshore pipelines are expected to withstand this level of stresses without excessive strains.

Over the last two decades, the ice scour problem, and in particular, subscour soil deformation has been tackled using different means based on the available tools at the time. Such studies include small and medium scale experimental testing, analytical and empirical formulations, simplified structural analyses, and advanced numerical techniques. A summary of publicly available subgouge soil deformation data is presented in Table 1.

Despite the work done to date, there remains uncertainty on the magnitude and extent of subgouge soil deformations, and thus, uncertainty in target burial depth requirements, which directly impacts development cost and safety. This paper presents a model that allows for direct and explicit estimation of pipe responses, and will be significant in optimizing the pipeline burial depth, where the pipeline integrity can be maintained with tolerable permanent deformation, at a reasonable cost. The developed model is used to examine the effect of keel angle, keel geometry (conical vs. rectangular), and scour depth on soil failure mechanisms and induced subgouge deformations.

DEVELOPMENT OF FE MODEL

Advancements in computational methods (computer hardware and software technology) have provided improved engineering tools to analyze complex nonlinear geotechnical failure processes, load transfer, and pipeline soil interaction events and pipeline failure mechanisms [Ref. 6]. Albeit, numerical modeling of large soil deformation employing complex constitutive models is a challenging problem. Konuk et al. [Ref. 8] reported several challenges in numerical modeling of ice gouge and pipeline response, such as proper understanding and modeling of the ice indenter, determination of initial geometry of the ice feature which gouges the seabed, selection or development of a constitutive soil model, selection of discretization method, definition of the interface processes, and contact mechanics between the ice ridge and seabed. Nevertheless, numerical formulations have proven the ability to model fully coupled ice/soil/pipe interactions with reliable results [Ref. 15].

Different numerical formulations have been used to model the gouging problem such as pure Lagrangian, Updated Lagrangian (UL), pure Eulerian, mesh-free, Arbitrary Lagrangian Eulerian (ALE), and others. The principle advantage of using a purely Lagrangian formulation is the precise definition of the interface between the ice ridge and subsoil (nodes are fixed within the material). However, this technique provided limited success due to mesh distortion and

convergence problems as reported by Woodworth-Lynas et al. [Ref. 18]. The use of Lagrangian analysis with re-meshing produced errors due to projecting the nodal variables to the nodes of new mesh. It is especially difficult to maintain mass conservation during the projections from the old mesh to the new mesh [Ref. 10].

Mesh-free methods are particular types of Lagrangian formulation with unique characteristics. They avoid the problem of mesh tangling since they do not employ explicit nodal connectivity. Thus, nodes are allowed to move about the domain in Lagrangian fashion and interactions between the nodes are determined as part of the computation. This determination of nodal interactions without an explicit mesh tends to make mesh-free methods computationally expensive [Ref. 4].

The ALE method was originally developed for solving problems related to fluid-structure interaction. During ALE analysis, mesh motion is constrained to the material motion only at free boundaries. Thus, mesh distortion is minimized by adjusting mesh within the material free boundaries. [Ref. 10] and [Ref. 9] used the ALE method to model ice scour in three steps:

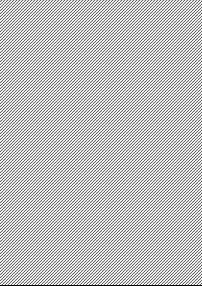
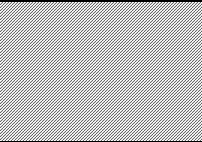
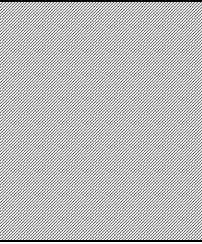
1. Conduct standard large deformation explicit Lagrangian FE analysis.
2. Remap the FE mesh based on smoothing criteria.
3. Compute discretized strain, mass, and momentum for each node of the new mesh using advection algorithm based on laws of conservation.

This “adaptive” meshing combines the features of pure Lagrangian analysis and pure Eulerian analysis. However, ALE adaptive meshing does not alter the topology (elements and connectivity) of the mesh, which implies some limitations on the ability of this method to maintain a high-quality mesh upon extreme deformation.

A detailed literature review on ice gouge formulations may be found in [Ref. 5] by the authors.

In this paper, a 3D FE model was developed to study some of the important aspects of the ice gouging process, and particularly, subgouge soil deformations. The model uses the CEL formulation available in Abaqus/Explicit software. The CEL formulation accommodates large deformations that occur during gouging by allowing Eulerian and Lagrangian bodies within the same model to interact, and at the same time, avoids any stability problems. During CEL analysis, the Lagrangian mesh interacts with the materials in the Eulerian mesh, with both meshes assembled in the same model. The interaction between Lagrangian surfaces and Eulerian material in the model is achieved using the “general contact” option. While Lagrangian and Eulerian elements and nodes can overlap, three-dimensional Lagrangian elements cannot occupy the same space as an Eulerian material instance. Therefore, Lagrangian parts must be instanced in an area of void within the Eulerian part instance. The volume fraction tool can be used to create a void

Table 1 Summary of Available Subgouge Soil Deformation Data for Clay Soils

Data Source	Test ID	Keel Angle (°)	Prototype Keel Width (m)	Prototype Gouge Depth (m)	Soil Type (Given)	Horizontal Subgouge Deformation Data at Gouge Centerline
Centrifuge Tests (Woodworth-Lynas et al., 1996)	PRISE 03-D1	15	15	1	Stiff Clay	Normalized average centerline horizontal subgouge deformation (idealized equation also presented).
	PRISE 03-D2			2		
	PRISE 05-D1	15	15	1	Soft Clay	
	PRISE 04-D1	30	30	1	Stiff Clay	
	PRISE 04-D2	15				
	PRISE 06-D1	30	15	1	Soft Clay	
Centrifuge Tests and accompanying 2D FEA (Lach, 1996)	Test 01	15	10	0.38	Speswhite Kaolin Clay (Cam-clay parameters + typical soil parameters)	Average centerline horizontal subgouge deformation in the apparent steady-state gouge region.
	Test 02			1.20		
	Test 04			1.46		
	Test 05			1.21		
	Test 06			1.63		
	Test 07			1.37		
	Test 09			1.10		
	Test 08	25	10	2.23		
3D ALE FE Model – Eulerian Option in LS-DYNA (Konuk et al., 2005)		45	15	1	Soft Clay (LS-DYNA Cap Model parameters)	Resultant location of centerline tracer particles originally placed in a vertical column at 30 m from model edge for six cases.
				2		
		30		1		
				2		
		15		1		
				2		
3D ALE FE Model – ABAQUS (Kenny et al., 2007)		15	10	1.46	Soft Clay ($S_u = 25$ KPa)	Resultant location of centerline tracer particles originally placed in vertical columns at three intervals along the gouge length.
2D FD ALE & 2D & 3D FE UL – PLAXIS (Liferov et al, 2007)		30	30	1	Stiff Clay	Average centerline horizontal subgouge deformation.
		15				
		30	15	1	Soft Clay	
		15	15	1		
		30	15	1		
Reduced Scale Laboratory Tests (Been et al., 2008)	Test 1	45	1.5	0.150	Soft Clay ($S_u = 15$ to 20 KPa)	Average centerline horizontal subgouge deformation (idealized equation also presented).
	Test 3	45	1.5	0.213		
	Test 5	15	1.5	0.110		
	Test 8	15	1.5	0.275		
	Test 10	30	1.5	0.150		

region within Eulerian material to be occupied later by a three-dimensional Lagrangian part [Ref. 1].

The model, presented schematically in Figure 1, consists of the following parts:

Eulerian domain: This domain encloses all materials and Lagrangian parts. In Abaqus, initial locations of different materials may be defined within the Eulerian domain using the Volume Fraction tool. The Eulerian domain labeled “VOID” in Figure 1 is divided into three parts: The initial seabed material, the initial trench backfill material, and absence of material (or void). Material displaced by the translation of the iceberg may then occupy the elements that were once assigned void in subsequent time steps. Void is also assigned to elements within the pipe. Elements not fully enclosed within the outside diameter of the pipe are assigned a volume fraction of void less than one. The Eulerian mesh is modeled using an 8-node linear brick, multi-material, reduced integration with hourglass control (linear hexahedral element type EC3D8R). The Eulerian domain is also extended to fully enclose the initial position of the ice ridge geometry.

Lagrangian pipe elements: The pipeline extends outside the seabed and trench material, but not beyond Eulerian domain. The pipe is modeled as three-dimensional deformable homogenous, doubly curved general-purpose shell using 4-node, reduced integration with hourglass control (linear quadrilateral element type S4R).

Ice ridge: The ice indenter is modeled as a three-dimensional solid rigid shape Lagrangian part that moves through the Eulerian mesh. Except for recent studies by Liferov et al. [Ref. 13], ice indenter has been reasonably presented as a rigid body in numerical modeling [Ref. 6, Ref. 14, and Ref. 15]. The motion of the ice ridge is determined by initial penetration depth, and a prescribed velocity. A general contact with penalty based friction is defined between the ice ridge and the soil. The ice ridge is modeled using 4-node linear tetrahedrons (C3D4). Varying values of base scour width, scour depth, geometry, attack angle, and speed may be assigned.

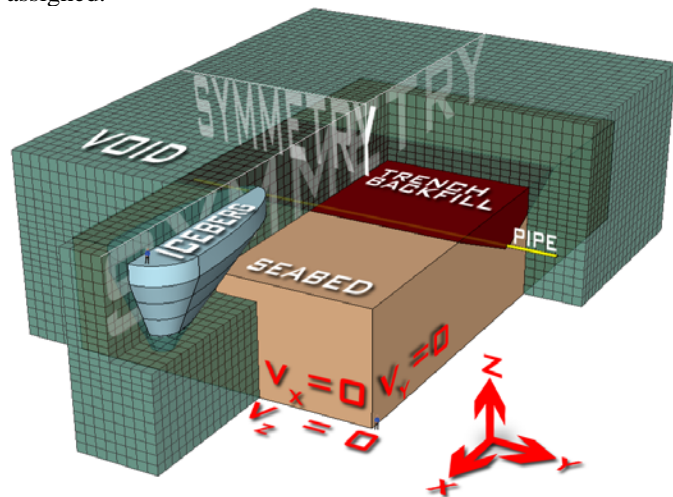


Figure 1 CEL FE Model Schematic

Analysis was conducted as a dynamic problem incorporating inertia effects, and was completed in three steps:

Step 1:

- Define the initial location of trench material and seabed soil;
- Define penetration depth of ice ridge. This parameter may be changed to investigate its effect on soil and pipeline deformation. The ice ridge is placed outside the soil but within the Eulerian domain;
- Define explicit general contact between ice ridge and seabed;
- Apply boundary conditions to the pipe; both ends of the pipe are restrained from translation and rotation in all directions.

Step 2:

- Apply body force that presents the submerged unit weight of seabed soil. The purpose of this gravity load is to match the stress state in the soil with the natural field conditions. Thus, it is crucial that sufficient time is allowed for the soil stresses to reach a stationary state. This load is continued to the next step;
- Restrain the Eulerian material from flowing outside the predefined dimensions through the application of velocity boundary conditions. The velocity/angular velocity was set to zero in the perpendicular direction at the bottom and all planar boundaries of the soil extending to the top of the Eulerian domain, while the top surface was left free to move within the void;
- Restrain the Ice ridge from movement in all directions.

Step 3:

- Define ice ridge motion as a translation in scour direction only. All other translational/rotational degrees of freedom are constrained. The ice ridge speed may be set to present expected field conditions.

Soil is modeled by the modified Drucker-Prager/Cap constitutive model. Generally, this model is used to simulate the constitutive response of cohesive materials. It adds a “cap” yield surface to the linear Drucker-Prager model to bound the material in the hydrostatic compression and to help control volume dilatancy when the material yields in shear. The transition between the Drucker-Prager cone and the cap is quite important in providing the desired realistic and smooth response from the deforming mass of soil that is subject to both large plastic shear deformation as well as large pressure. The soil is modeled as a single-phase material, presenting undrained conditions. Although it is desirable to incorporate a two-phase model to capture the change in pore pressure due to effective stress change, Palmer et al. [Ref. 15] reported that gouging deformations in clayey soils may often occur in undrained conditions.

MODEL VERIFICATION

Generally, validation and/or calibration of newly developed models can be performed according to one or more of the following:

1. Comparison against full scale test results:

Full scale test results are not available in the literature. A full scale test is not expected to take place in the near future as these tests are prohibitively expensive. Moreover, Schoonbeek et al. [Ref. 17] argued that a real scale test, although may appear to be the most reliable way of examining a particular problem, it will be an enormous project involving huge and unwieldy loads that are hard to control accurately in the case of ice scour. They added that the need to examine a variety of boundary conditions will add to the cost and raises the issue of reproducibility.

2. Comparison against small scale and medium scale experimental testing:

These include physical tests carried out in geotechnical centrifuges, and reduced scale laboratory tests. The main advantage of these methods, unlike full scale tests, is that they can be accurately reproduced to examine the effect of slight design differences [Ref. 17]. Small scale laboratory model tests however, cannot be expected to provide results directly applicable to field situations, as similitude is not achieved due to unrealistic soil stresses. Centrifuge testing improves the representation of soil behavior as the applied gravity loading provides attainment of similar stress conditions to the prototype [Ref. 11].

3. Comparison against analytical models, empirical relations, structural models, and other numerical models that has been validated/calibrated:

The ice scour process is too complex to be accurately represented by simple mathematical formulations. Instead, empirical/semi-empirical formulations of subgouge displacements based on centrifuge tests [Ref. 18] and small scale laboratory experiments [Ref. 3] exist in the literature.

The structural spring formulations, defining soil resistance to deformations in two- or three-dimensional space, are usually assumed to be independent and the responses are discrete between adjacent soil zones. This idealization does not truly replicate a soil medium behavior [Ref. 14].

Proprietary finite element models have been developed and comparisons of subgouge soil deformations have been made with empirical relations and/or results from reduced scale centrifuge tests. Examples including Konuk et al. [Ref. 9, Ref. 10], Palmer et al. [Ref. 15], Kenny et al. [Ref. 6] and Liferov et al. [Ref. 13] are summarized in Table 2. However, with the extensive list of parameters involved, lack of published input data, and the observation of the subgouge comparison results, a straightforward interpretation of the validation analyses is not possible.

Table 2 Summary of the Developed FE Models and their Validation

Author	Comparison Test ID
Palmer et al. (2005) – Model described in Konuk et al. (2004 & 2005)	PRISE Function (Woodworth-Lynas et al., 1996)
Kenny et al. (2007)	Test 04 (Lach, 1996) and PRISE Engineering Model (Woodworth-Lynas, 1996)
Liferov et al. (2007)	PRISE 04-D1, 04-D2, 06-D1 (Woodworth-Lynas et al., 1996) & Konuk et al. (2005) 15° and 30° keel angle at 1 m gouge depth

In this paper, the results from the developed CEL model were compared to selected PRISE centrifuge experimental data and the empirical PRISE function [Ref. 18], to the data of centrifuge testing performed by Lach [Ref. 11], and to results presented by two FE models: Konuk et al. [Ref. 9] and Kenny et al. [Ref. 6].

Comparisons were limited to subgouge predictions in soft clay. In order to reduce the complexity of the problem and to match numerical, empirical and experimental conditions in the scour process, the trench geometry, trench backfill material, and the presence of the pipeline were avoided in the subgouge soil displacement predictions. The simplified model is presented in Figure 2. A typical FE output showing the scour process is presented in Figure 3. It should be noted that in comparison with Konuk’s model, the soil parameters were matched as well as scour depth and base width, keel geometry, attack angles, and scour speed. In comparisons with the model developed by Kenny et al. [Ref. 6] and PRISE test results, reasonable assumptions had to be made for some soil parameters due to limited availability of soil data. All other parameters related to ice indenter were matched. A comparison matrix is presented in Table 3.

Table 3 CEL FE Comparison Matrix

Data Source	Keel Shape	Keel Angle (°)	Keel Width (m)	Gouge Depth (m)	Keel Speed (m/s)	Soil Type
F.E. Model (Konuk et al., 2005)	Conical	45	15	2	1	Soft Clay
	Conical	30	15	2	1	Soft Clay
	Conical	15	15	2	1	Soft Clay
Centrifuge Test (Woodworth-Lynas et al., 1996)	Rectangular	30	15	1	0.1	Soft Clay
	Rectangular	15	15	1	0.1	Soft Clay
F.E. Model (Kenny et al, 2007)	Rectangular	15	10	1.46	0.1	Soft Clay

Comparing the current FE predictions in Figure 4 and Figure 5 in relation with the PRISE Model may initially give an impression of inconsistent results, but one should keep in mind that these predictions represent two different loading conditions, which will be addressed in the sensitivity study later in this paper.

Figure 6 shows a comparison of the results obtained by the present study to those presented by Kenny et al. [Ref. 6] using an ALE FE model and experimental data from a centrifuge test performed by Lach [Ref. 11]. It is shown that the current predictions are in agreement with predictions by Kenny et al. When compared to the PRISE Model, the CEL FE yielded higher displacement immediately below scour depth and within one scour depth. The predictions agreed well with the experimental data (Lach) at depths greater than one to two gouge depths.

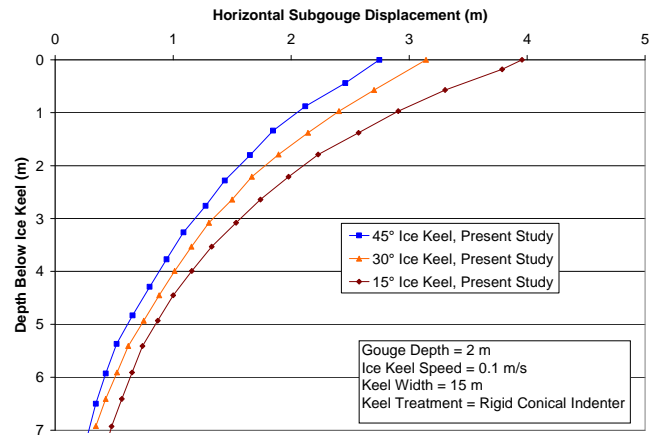


Figure 7 Effect of Keel Angle on Subgouge Deformations

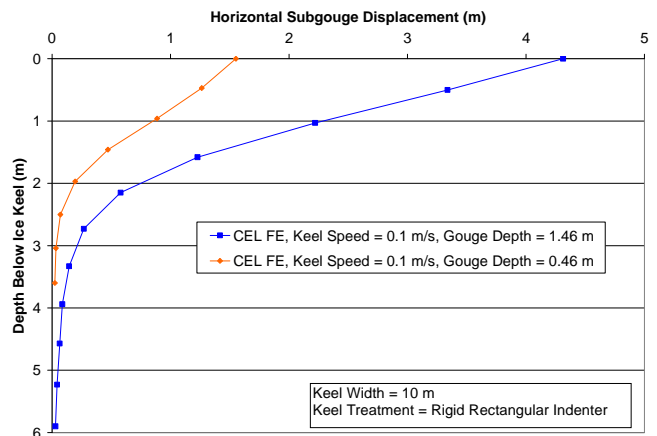


Figure 8 Effect of Gouge Depth on Subgouge Soil Deformations

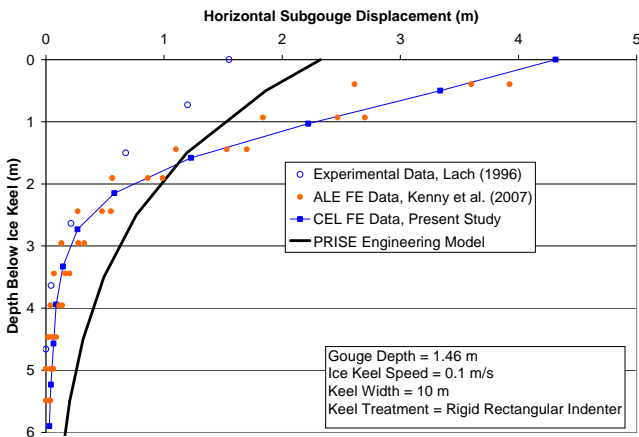


Figure 6 Comparison of Horizontal Subgouge Deformations Predictions between CEL FE and Kenny et al. [Ref. 6]

SENSITIVITY STUDIES

The developed model was used to examine the effect of keel configurations (depth, angle, and geometry) on the induced horizontal subgouge deformations. The effect of gouge width was not explicitly addressed in the present study however previous work suggests that subgouge deformations are less sensitive to this parameter as keels become wider [Ref. 3].

Figure 7 shows the effect of keel angle on the subgouge displacements. These results correspond to a gouge depth of 2 m, scour speed of 1 m/s, and a conical shape keel. The soil is soft clay. It can be seen that shallower keel angles produce larger subgouge soil displacements, and may be more critical in design considerations.

Figure 8 shows the effect of gouge depth on the horizontal subgouge soil displacement. It has been shown and may be concluded that for the same soil, deeper gouging keels produce larger subgouge soil deformations, as this is a consistent trend in the available literature.

With respect to soil failure and mound formations, it was observed in Figure 9 that during the gouging process, and for low keel angles (15°), the soil moves sideways beyond the edge of the keel and then upwards. The soil is compressed below the keel, and then squeezed upwards, forming a low mound along the gouge sides. This side mound is higher in conical shape keels than in rectangular keels. On the other hand, for higher keel angles (30° and 45°) the soil is lifted upwards ahead of the keel forming a mound immediately in front of the keel, which will be subsequently cleared to the gouge side. The berm formed along the gouge sides is steeper-sided and higher compared to that of lower keel angle. Again, the front and side mounds are higher in conical keels than rectangular ones. In addition, it appears that the sharp edges of a rectangular keel produce higher stresses in the soil mass just beneath the keel-soil contact in comparison to a conical keel.

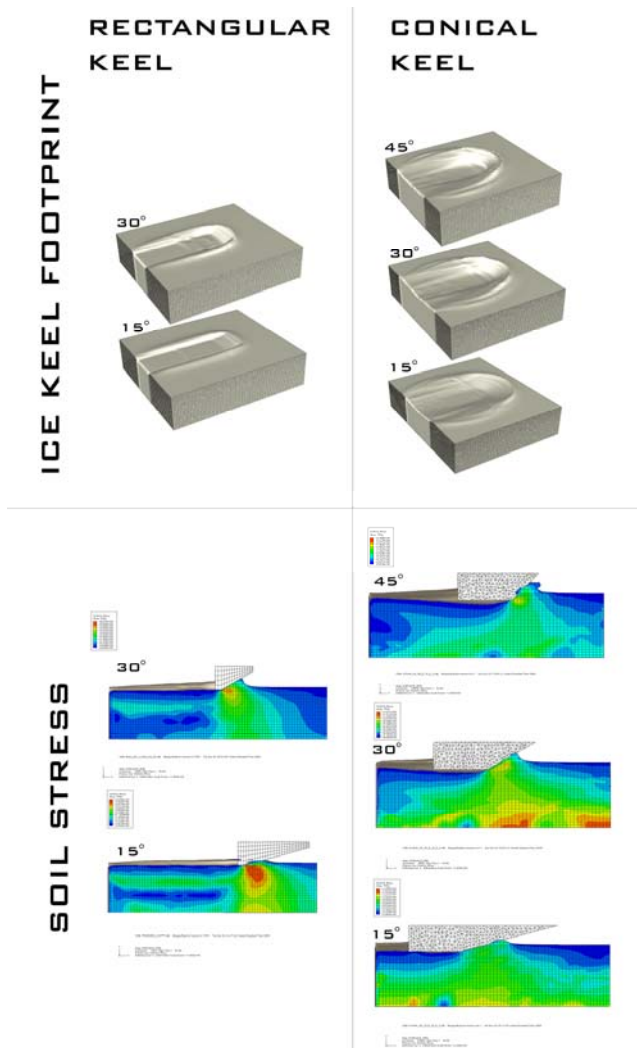


Figure 9 Effect of Keel Shape and Angle on Soil Failure and Stress (soil stresses are shown for comparison purposes only)

CONCLUSIONS

With the world's demand for energy ever increasing and with the arctic holding as much as a quarter of the world's untapped reserves of hydrocarbons, the move towards developing the Arctic hydrocarbon resources appears inevitable. Laying pipelines in these regions, however, is still an evolving technology, and the involved challenges point out to the need for new standards and industry practices. Of particular significance is the issue of pipeline burial depth to meet safety target levels. The CEL FE technique described in this paper is ideally suited to numerically represent the ice gouge process. The most challenging aspect in applying this technology is modeling the soil behavior. For design applications, uncertainty in soil response may be reduced by performing soil tests required to determine the parameters of the applied constitutive model.

A detailed description of a CEL FE model for determination of soil displacement and subsequent pipeline response is given. A reduced complexity model is then presented in order to compare results of centerline horizontal

subgouge deformation to the summarized publicly available data.

Results are compared with data from centrifuge tests performed during the PRISE study as well as with FE models developed by Kenny et al. [Ref. 6] and Konuk et al [Ref. 10]. Results agreed particularly well with FE data provided by Kenny et al. [Ref. 6] and likewise, the predicted subgouge horizontal soil displacements were larger up to one gouge depth and then tended towards experimental centrifuge data with increasing depth.

The PRISE centrifuge data presented by Woodworth-Lynas et al. [Ref. 18] does not extend adequately to indicate smaller soil displacements in underlying zones and inaccuracy is introduced as the data for all plots except PRISE 06-D1 has been normalized; subgouge deformation plots must be estimated using fractions of the maximum displacement which may be calculated using the idealized PRISE Engineering Model. When compared to the PRISE data, the FE model predicted higher soil displacements immediately below the gouge base and within one to two scour depth, but smaller soil movement below this region. On average, the FE results generally agreed with the PRISE Engineering Model.

In relation to FE data presented by Konuk et al. [Ref. 9] the present study showed close agreement for the shallow 15° keel, however, results for the 30° and 45° keels did not exhibit good correspondence. Interestingly, the PRISE Engineering Model overlain on the plot agrees quite well with the results of the present study.

Preliminary examination of the sensitivity of parameters related to the magnitude and extent of subgouge deformation showed the effects of altering gouge depth and keel angle. Results indicated that soil displacements were shown to be directly proportional to gouge depth and decreasing keel angle.

In closing, the CEL FE method conforms to the requirements of an accurate ice gouge model. The present study has shown that the developed model can reasonably predict horizontal subgouge deformation based on current available data. As progress has been made in predicting system demand, re-introduction of a pipeline to the model can be carried out with an attained level of confidence. The CEL FE method is a reliable engineering tool to model the ice gouge process.

REFERENCES

1. Abaqus Version 6.7 Extended Functionality Documentations, 2007.
2. Abdalla, B., Jukes, P., Eltaher, A., and Duron, B., 2008, "The Technical Challenges of Designing Oil and Gas Pipelines in the Arctic" Oceans'08 MTS/IEEE conference, Quebec City, Quebec, Canada.
3. Been, K., Sancio, R. B., Ahrabian, D., Kesteren, W., Croasdale, K., and Palmer A., 2008, "7th International Pipeline Conference, Calgary, Alberta, Canada.
4. Brown, K. H., Burns, S. P. and Christon, M. A, 2002, "Coupled Eulerian Methods for Earth Penetrating Weapon Applications," A Report prepared by Sandia National Laboratory.
5. Jukes, P., Eltaher, A., Abdalla, B. and Duron, B., 2008, "The Design and Simulation of Arctic Subsea Pipelines – Ice Gouging Formulations," 4th annual Arctic Oil & Gas conference, Oslo, Norway.
6. Kenny, S., Barrett, J., Phillips, R. and Popescu, R., 2007, "Integrating Geohazard Demand and Structural Capacity Modeling within a Probabilistic Design Framework for Offshore Arctic Pipelines," Proceedings of the 17th International Offshore and Polar Engineering Conference, Lisbon, Portugal.
7. Kenny, S., Palmer, A.C. and Been, K., 2007, "Design challenges for offshore pipelines in arctic environments." Proc., Offshore Oil and Gas in Arctic and Cold Waters, 15p.
8. Konuk, I., Liferov, P., & Loset, S., 2007, "Challenges in modelling ice gouge and pipeline response." Port and Ocean Engineering Under Arctic Conditions (POAC), Dalian, China.
9. Konuk, I., Yu, S. and Garcia, R., 2005, "A 3-Dimensional Continuum ALE Model for Ice Scour – Study of Trench Effects," OMAE'05 24th International Conference on Offshore Mechanics and Arctic Engineering, Halkidiki, Greece.
10. Konuk, I., Yu, S., and Garcia, R., 2005, "An ALE FEM Model of Ice Scour," 11th International Conference of the International Association of Computer Models and Advances in Geomechanics, Turin, Italy.
11. Lach, P.R., 1996, Centrifuge Modeling of Large Soil Deformation due to Ice Scour. Doctoral Thesis, Memorial University of Newfoundland.
12. Lach, P. R. and Clark, J. I., "Ice Scouring of the Seabed," A Research Paper prepared for Natural Sciences and Engineering Research Council.
13. Liferov, P., Shkhinek, K. N., Vitali, L., & Serre, N., 2007, "Ice gouging study - actions and effects." Port and Ocean Engineering Under Arctic Conditions (POAC), Dalian, China.
14. Nobahar, A., Kenny, S. and Phillips, R., 2007, "Buried Pipelines Subject to Subgouge Deformations," International Journal of Geomechanics, ASCE, Volume 7, No. 3.
15. Palmer, A. C., Konuk, I., Niedoroda, A.W., Been, K. and Croasdale, K. R., 2005, "Arctic Seabed Ice Gouging and Large Sub-Gouge Deformations," 18th International Conference on Port and Ocean Engineering under Arctic Conditions (POAC), Potsdam.
16. Palmer, A., Konuk, I., Love, J., Been, K. and Comfort, G., 1989, "Ice Scour Mechanics," A Research Paper prepared for Canada Oil and Gas Lands Administration and Gulf Canada Resources Ltd.
17. Schoonbeek, I.S.S., and Allersma, 2006, "Centrifuge Modeling of Scouring Ice Keels in Clay". Physical Modeling in Geotechnics – 6th ICPMG'06 – Ng, Zhang & Wang (eds).
18. Woodworth-Lynas, C., Nixon, D., Phillips, R., and Palmer, A., 1996, "Subgouge Deformations and the Security of Arctic Marine Pipelines," Proc. 28th OTC, Vol. 4, Paper No. 8222, pp. 657–664.